647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov. - 2023(Old Course)
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one out of two. (07)

1. State and prove Housed Orff property.
2. X is metric space. Show that union of finite numbers of closed subsets of X is closed. Explain with example that this statement is not true for infinite numbers of closed subsets of X.

Que-1(B) Answer any one out of two. (04)

1. Find $E^0, E', \overline{E}$ where E={1,2,3,4,5,…………..}.
2. (i) R is discrete metric space. Find N(2,0.1) in R.
   (ii) Construct set of real numbers which has exactly Two limit points.

Que-1(C) Answer any one out of two. (03)

1. Show that cantor set is closed and prove that $\frac{3}{4} \in$ C.
2. E=(0,1). Is E closed? E is open? Explain your answer.

Que-2(A) Answer any one out of two. (07)

1. prove that any bounded function f defined on [a,b] is R-integrable iff For ∀ ε>0 there exist partition P of [a,b] such that U(P,f) - L(P,f) < ε , Where $\|P\| < \delta$.
2. if f $\in$ $R_{[a,b]}$ then for $\forall \lambda \in$ R show that λ f $\in R_{[a,b]}$.

Que-2(B) Answer any one out of two. (04)

1. Find $\int_0^1 x^2 \, dx$ using definition of R-integration.
2. If bounded function f on [a,b] is monotonic then show that f is R-integrable.

Que-2(C) Answer any one out of two. (03)

1. Using darbour's theorem find value of
   $$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots\ldots\ldots\ldots + \left(\frac{n}{n}\right)^2\right]$$
2. Let F(x)=k where k $\in$ R and x $\in$ [a,b]. prove that F(x) $\in R_{[a,b]}$.

Que-3(A) Answer any one out of two. (07)

1. if bounded function f is R-integrable in [a,b] and F(x)= $\int_a^x f(t)\,dt$, Where a≤x≤b then show that F(x) is continuous in [a,b].
2. State and prove general form of first mean value theorem of integral calculus.

Que-3(B) Answer any one out of two. (04)

1. if a,b $\neq$ 0, a>b then show that $\frac{\pi^2}{2a} < \int_0^{\pi} \frac{x}{a\cos^2\frac{x}{2} + b\sin^2\frac{x}{2}}\,dx < \frac{\pi^2}{2b}$.
2. Binary operation * is associative in non-empty set A. a,b ∈ A are non-singular elements of A. prove that a*b is also nonsingular element of A.

Que-3(C) Answer any one out of two. (03)

1. G=R-{1}. Binary operation * is defined on G as; for a,b G , a*b=a+b-ab then show that (G,*) is group.
2. For f(x)= x, x [0,1] and partition P={0, $\frac{1}{n}$, $\frac{2}{n}$, $\frac{3}{n}$,……. $\frac{n}{n}$ }. Find L(P,f) and U(P,f).

Que-4(A) Answer any one out of two. (07)

1. State and prove lagrange's theorem.
2. Prove that order of permutation f Sn is L.C.M. of length of disjoint cycles of f.

Que-4(B) Answer any one out of two. (04)

1. G={(a,b)/a,b$\in$R and a$\neq$0}and binary operation * define on G as:
   For a,b,c,d $\in$R, (a,b)*(c,d)=(ac,bc+d). Show that if H={(1,b)/b$\in$R} then H $\leq$G. Where (G,*) is group.
2. $f \epsilon$ S_6 where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 5 & 4 & 2 & 1 \end{pmatrix}$ then find f^{-1} and O(f). Also Check whether f is even or odd.

Que-4(C) Answer any one out of two. (03)

1. G is group and H≤G. then prove that any two left coset of H in G is either equal or disjoint.
2. Show that set of all qube roots of 1forms a group under multiplication and find order of elements of this group.

Que-5(A) Answer any one out of two. (07)

1. if (G,*) ≅ (G',*'). Show that G is commutative iff G' is commutative.
   Where G and G' are groups.
2. State and prove cayley's theorem.

Que-5(B) Answer any one out of two. (04)

1. G is group and H≤G. if index of H in G is 2 then prove that H is normal Subgroup of G.
2. H and K be two normal subgroups of group G. if H$\cap$K={e} then for $\forall$ h$\in$H and k$\in$K prove that hk=kh·

Que-5(C) Answer any one out of two. (03)

1. find numbers of generators of cyclic group Zpq. Where p and q are prime Numbers.
2. Prove that for finite cyclic group, order of group is same as order of it's
3. generator.

****